

Quasigeoid for North Macedonia

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1 Introduction

This document is an update of the document dated 14. December 2020. The connection points has been updated (H_{normal}). A quasigeoid covering North Macedonia based on available gravity data is estimated. The resulting quasigeoid will be compared to an updated 27 connection points (combined GNSS/leveling points).

2 Data

A short description of the data used in estimation of the quasigeoid will be given in this chapter.

2.1 Digital terrain grid

A digital terrain model (DTM) extending in latitude from 39.7 to 43.6 degrees and in longitude from 19.1 to 24.4 degrees was extracted from European Digital Elevation Model (EU-DEM), version 1.1 (<https://land.copernicus.eu/imagery-in-situ/eu-dem/eu-dem-v1.1>). It has a 25 m resolution with vertical accuracy: +/- 7 meters RMSE. Three DTMs with spacing 50 m, 1000 m and 5000 m were used in the geoid computation.

Initially, a 50 m terrain grid covering only the territory of North Macedonia where provided by AREC, hereafter called AREC-DEM. A comparison of the two grids (AREC-DEM and EU-DEM), show differences up to 500 meters, see Table 1 for statistics. There are areas where the two models have large differences, for instance in Lake Ohrid. Figure 1 shows the area covering Lake Ohrid, where the AREC-DEM show features deviating from an assumed constant lake surface at 695 meters. The EU-DEM give a constant height values of 688 meters for most part of Lake Ohrid, which is close to the official value.

We ended up using the EU-DEM as terrain data from neighboring countries are needed.

| Model | Min | Max | Mean | SD |
|------------|---------|---------|--------|--------|
| EU-DEM | 14.66 | 2714.66 | 820.95 | 465.13 |
| AREC-DEM | 32.11 | 2738.50 | 828.82 | 443.51 |
| Difference | -294.55 | 492.90 | -1.35 | 16.09 |

Table 1: Statistics of the terrain model EU-DEM and AREC-DEM and the difference between the two models. All values in meters.

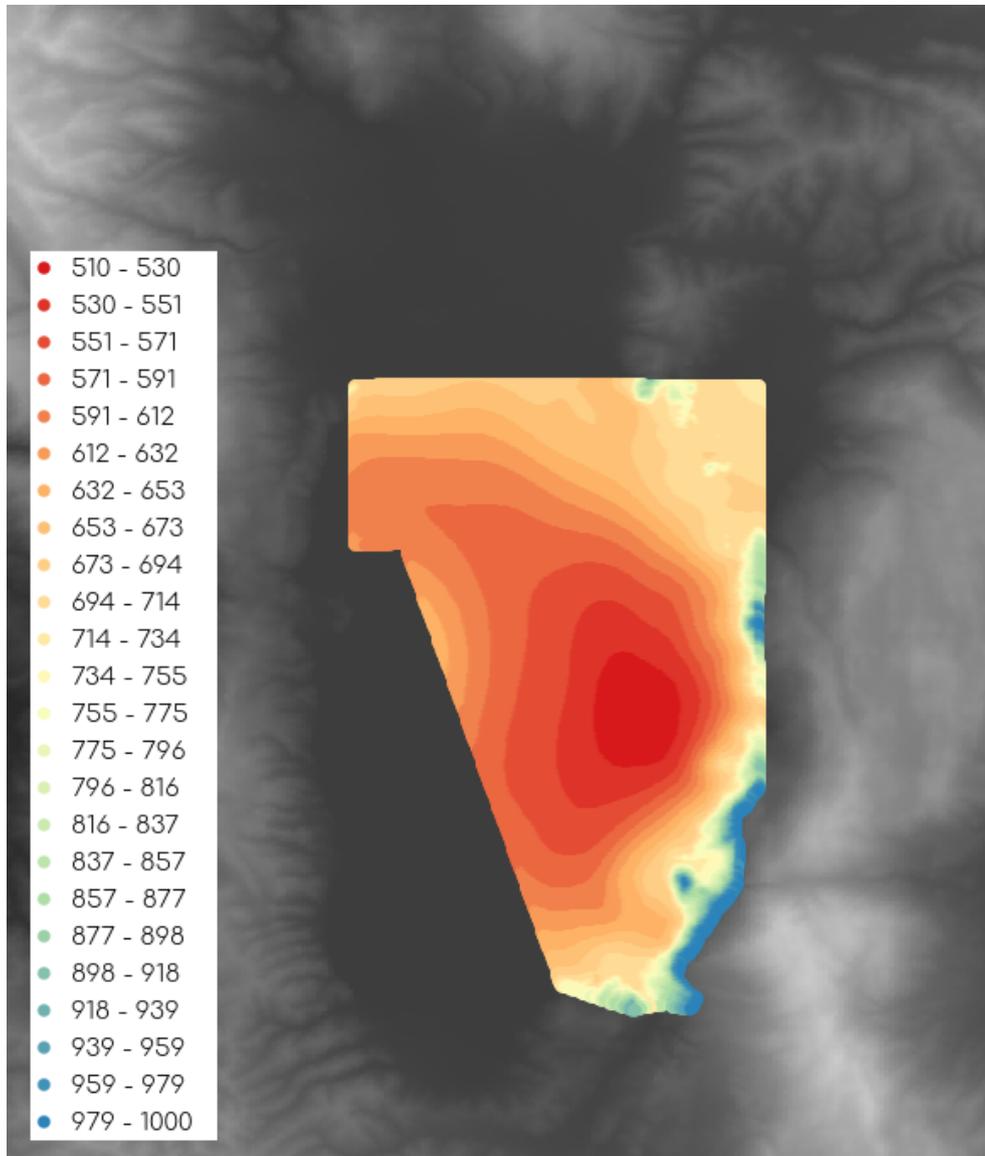


Figure 1: The terrain covering Lake Ohrid. In colors are the AREC-DEM and in shades of gray are the EU-DEM. EU-DEM show a constant values across Lake Ohrid, while AREC-DEM deviates from an expected constant surface. Values in meters.

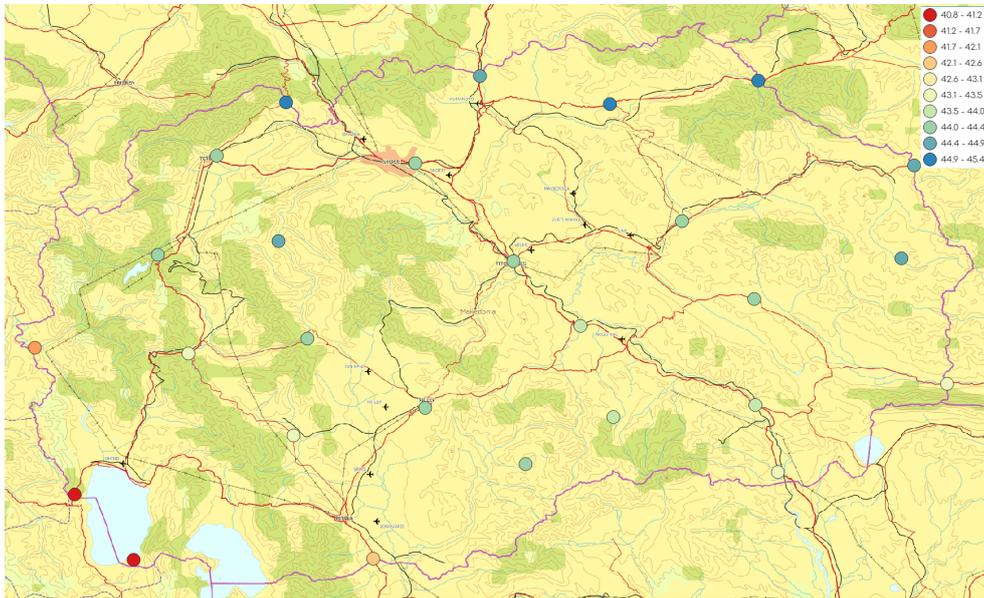


Figure 2: GNSS/leveling data (H_{nvt1}) as circles. Values in meters.

2.2 Gravity data

The following gravity data has been provided by AREC:

- 36 points from Basis gravity network; ‘Basic Gravity Network - DEF.xlsx’. Removed all duplicate data points.
- 27 GNSS leveling points w gravity; ‘GNSS leveling data - DEF.xlsx’
- 3 updated GNSS leveling points. Changes in both H_{nvt1} and H_{normal} ; ‘New GNSS leveling data for suspicious points.xlsx’
- 13 points compared to MAKPOS; ‘GNSS leveling data - differences with MAKPOS and without MAKPOS.xlsx’
- 1178 leveling/gravity points; ‘Leveling network - DEF.xlsx’
- 159 passive GNSS points w gravity; ‘Passive GNSS network - DEF.xlsx’
- 1054 gravity points; ‘GRID points - DEF.xlsx’

2.3 Combined GNSS/Leveling points

There are 27 combined GNSS and leveling points, they are given as circles in Figure 2. Three of the points have been quality controlled and updated. The GNSS points are given in the ETRS89 reference frame, with epoch 1989.0.

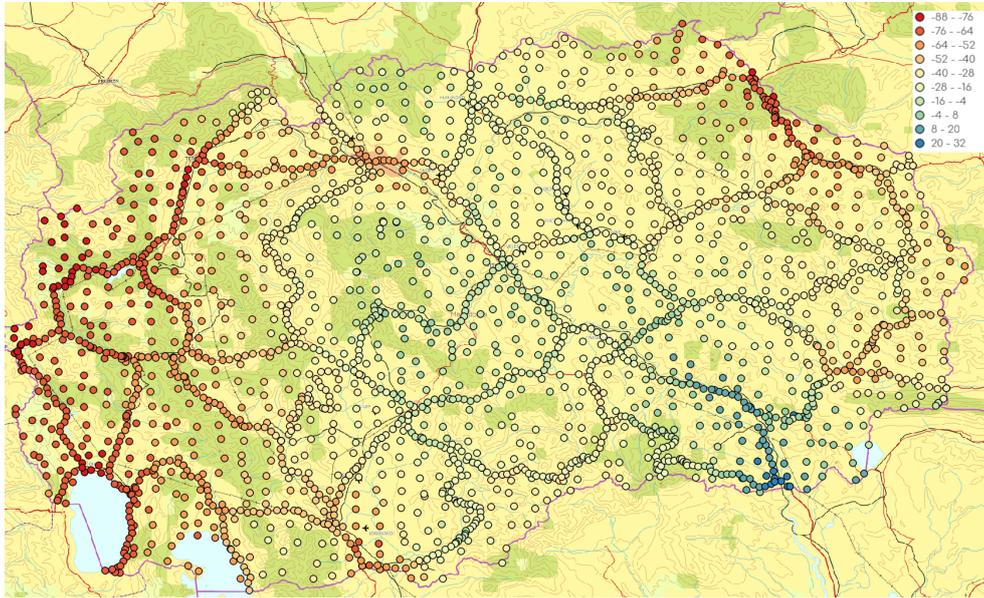


Figure 3: Gravity data (Bouguer anomalies) covering North Macedonia. Values in mGal.

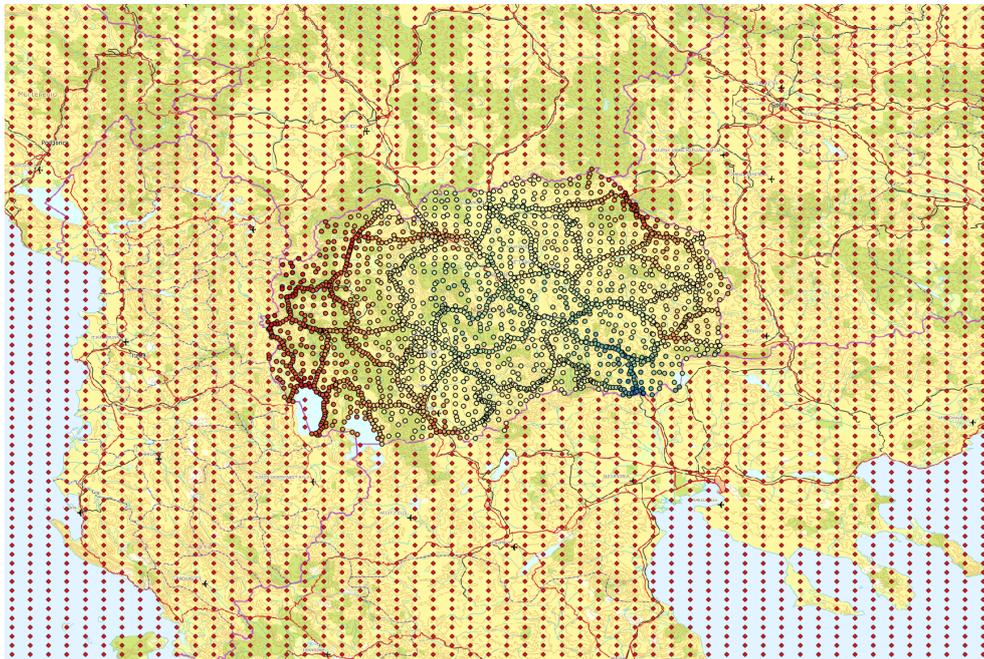


Figure 4: Gravity data covering North Macedonia (circle) and extra gravity data from EGM08 (red diamond)

2.4 Height system

The official height system is H_{nvt1} and the new unofficial height system is H_{normal} . Both system uses normal heights.

The height system H_{nvt1} :

- also referred to as the NVT High Precision Leveling Network 1
- normal orthometric heights based on the high-precision leveling conducted in the period 1946 - 1957.
- The vertical reference level of the NVT1 heights coincides with the mean level of the mareographic measurements carried out on the mareograph in Trieste in 1875.

The height system H_{normal}

- also referred to as NVT2
- normal orthometric heights based on the high-precision leveling conducted in the period 1968 - 1973
- The vertical reference level to the heights is the mean level of mareographic measurements conducted on mareographs in Kopar, Rovinj, Bakar, Split, Dubrovnik and Bar, along the Adriatic Sea, from 1962 to 1980 (epoch 1971.5).

More information about the height systems are given in Law for Real Estate Cadastre (Official Gazette of the Republic of Macedonia” No. 55/2013, 41/2014, 115/2014, 116/2015, 153/2105, 192/2015, 61/2016, 172/2016, 64/2018 and ”Official Gazette of Republic of Northern Macedonia ”No. 124/2019) and the Sub-law for basic geodetic works (Official Gazette of the Republic of Macedonia, no. 151 dated 04.11.2013).

3 Geoid computation

We use the remove-restore method in combination with the Residual Terrain Model (RTM) method (*Forsberg, 1984*) in computation of the quasigeoids. We follow the the RTM/Helmert method, as described in (*Omang and Forsberg, 2000*).

We first remove the global geopotential model (Δg_{ggm}) and the RTM effect of the terrain on gravity ($\Delta g_{terr-eff}^{rtm}$).

$$\Delta g^{rtm} = \Delta g_{fa} - \Delta g_{ggm} - \Delta g_{terr-eff}^{rtm} \quad (1)$$

where Δg_{fa} is the free-air anomaly, given as a point value at the surface of the topography. The RTM terrain effect on gravity is given as

$$\Delta g_{terr-eff}^{rtm} = 2\pi G\rho(h - h_{ref}) - c_p \quad (2)$$

where ρ is the density, h is the height of the topography and h_{ref} is the height of the smooth reference surface, e.g. of resolution 5 km.

Next, the RTM-reduced gravity data Δg^{rtm} are gridded. Before applying the Molodensky's formula we restore the terrain effect $2\pi G\rho(h - h_{\text{ref}})$, implying $\Delta g_{\text{terr-eff}} = -c_p$ in Eq (1).

In the RTM-method we use the free-air anomaly, Δg_{fa} , and Eq. (1), which refers to the surface of the topography, and we use the Molodensky integral which yields the residual quasigeoid height (height anomaly)

$$\zeta_{\Delta g} = \frac{R}{4\pi\gamma} \iint \left(\Delta g_{\text{fa}} + c_p \right) S(\psi) \cos \phi d\phi d\lambda \quad (3)$$

where ϕ, λ are geographical coordinates, γ is the normal gravity at the surface. The final quasigeoid is obtained by

$$\zeta = \zeta_{\text{ggm}} + \zeta_{\Delta g} \quad (4)$$

where $\zeta_{\Delta g}$ is the residual height anomaly. There is no terrain effect to restore since we added it back before applying the Molodensky formula.

We have estimated two versions of the quasigeoid, one with only gravity data from North Macedonia (version 1) and version 2 including extra gravity data (EGM08) covering an 150 km area surrounding North Macedonia.

3.1 Version 1

- Gravity data from North Macedonia, see Figure 3.
- As Global Geopotential Model (GGM): XGM2019 (Zingerle et al, 2019) up to degree and order 2190.
- A Stokes kernel with truncation of e.g. 140 with a linear taper to 160, indicating that below 140 we trust the global model while above degree 160 we trust our local data (100 and 120 for the EGM08 model). These truncation values gave the best fit between the estimated quasigeoid and the connection points.
- Terrain Correction is estimated with 50 m DTM (EU-DEM).
- To compute the RTM effect on the height anomaly, a DTM with spacing 1000 m and 5000m was used.

3.2 Version 2

- Same as version 1, however
- Additional gravity data from EGM08 (*Pavlis et al., 2012*) in a 150 km area around North Macedonia, see Figure 4.

4 Results

We compare the computed gravimetric quasigeoid to quasigeoid heights from GNSS and leveling data. We have used our two quasigeoid versions and global geopotential geoid models in the comparison.

- Generally
 - Our local quasigeoids give better results than the global models.
 - The updated GNSS/leveling points, in combination with global model XGM2019, give a huge improvement in the fit to the quasigeoids.
 - Adding gravity data from EGM08 do also give a significant improvement.
- Official height system: H_{nvt1}
 - Compared to the official height system using 27 points our best quasigeoid give a fit/std.dev. of 4 cm. See table 2.
 - See Table 4 and Figure 5 for the largest deviation.
- New height system: H_{normal}
 - Compared to the new height system using 27 points our best quasigeoid give a fit/std.dev. of 4.7 cm. See table 3.
 - See Table 5 for the largest deviation.

5 Conclusions

The updated GNSS/leveling data set gave a large improvement of the fit, and in combination with an improved global model (XGM2019) (*P. Zingerle, 2020*) we get a fit at the 4 cm standard uncertainty level (1 sigma).

Extracting gravity data from EGM08 (*Pavlis et al., 2012*) in a 150 km area around North Macedonia gave a significant improvement, indicating that gravity data from neighboring countries will give an improvement of the geoid.

References

Forsberg, R. (1984), A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modelling, *Tech. Rep. 355*, Dept of Geodetic Science and Surveying, Ohio State University, Columbus.

| Geoid | Min | Max | Mean | SD | Detrend | Comment |
|-------------|-------|-------|-------|-------|---------|-----------|
| Go_Dir_R6 | -0.92 | 0.34 | -0.32 | 0.32 | 0.29 | GGM |
| nmac_v1_gr6 | -0.54 | 0.12 | -0.11 | 0.18 | 0.15 | 140 - 160 |
| nmac_v2_gr6 | -0.33 | 0.05 | -0.17 | 0.081 | 0.073 | 140 - 160 |
| EGM08 | -0.51 | 0.11 | -0.21 | 0.16 | 0.10 | GGM |
| nmac_v1_egm | -0.40 | 0.02 | -0.15 | 0.086 | 0.060 | 100 - 120 |
| nmac_v2_egm | -0.34 | 0.01 | -0.15 | 0.087 | 0.046 | 100 - 120 |
| XGM2019 | -0.40 | -0.04 | -0.21 | 0.093 | 0.083 | GGM |
| nmac_v1_xgm | -0.32 | -0.08 | -0.16 | 0.060 | 0.056 | 140 - 160 |
| nmac_v2_xgm | -0.25 | -0.10 | -0.17 | 0.039 | 0.0378 | 140 - 160 |

Table 2: Statistics for the difference between different quasigeoids and 27 GNSS/Leveling points of the height system H_{nvt1} . As an example of the results see figure 5. All values in meters. The "Comment" column contains global model (GGM) and what level of truncation is used (e.g. 100 - 120). The "Detrended" column has a 4-parameter trend surface removed (bias and tilt).

Omang, O. C. D., and R. Forsberg (2000), How to handle topography in practical geoid determination: three examples, *Journal of Geodesy*, *74*, 458–466.

P. Zingerle, T. G. . X. O., R. Pail (2020), The combined global gravity field model xgm2019e, *94*(66), doi:<https://doi.org/10.1007/s00190-020-01398-0>.

Pavlis, N. K., S. A. Holmes, S. C. Kenyon, and J. K. Factor (2012), The development and evaluation of the earth gravitational model 2008 (egm2008), *Journal of Geophysical Research: Solid Earth*, *117*(B4), doi: <https://doi.org/10.1029/2011JB008916>.

| Geoid | Min | Max | Mean | Stddev | Detrended | Comment |
|--------------|-------|------|------|--------|-----------|-----------|
| Go_Dir_R6 | -0.58 | 0.64 | 0.02 | 0.32 | 0.28 | GGM |
| nmac_v1_gor6 | -0.22 | 0.46 | 0.22 | 0.18 | 0.15 | 140 - 160 |
| nmac_v2_gor6 | -0.01 | 0.37 | 0.16 | 0.081 | 0.072 | 140 - 160 |
| EGM08 | -0.20 | 0.44 | 0.11 | 0.16 | 0.11 | GGM |
| nmac_v1_egm | -0.08 | 0.33 | 0.18 | 0.084 | 0.060 | 100 - 120 |
| nmac_v2_egm | -0.02 | 0.34 | 0.18 | 0.087 | 0.051 | 100 - 120 |
| XGM2019 | -0.09 | 0.29 | 0.12 | 0.10 | 0.093 | GGM |
| nmac_v1_xgm | 0.03 | 0.25 | 0.17 | 0.061 | 0.054 | 140 - 160 |
| nmac_v2_xgm | 0.06 | 0.23 | 0.16 | 0.045 | 0.043 | 140 - 160 |

Table 3: Statistics for the difference between different quasigeoids and 27 GNSS/Leveling points of the height system H_{normal} . The "Comment" column contains global model (GGM) and what level of truncation is used (e.g. 100 - 120). The "Detrended" column has a 4-parameter trend surface removed (bias and tilt). All values in meters.

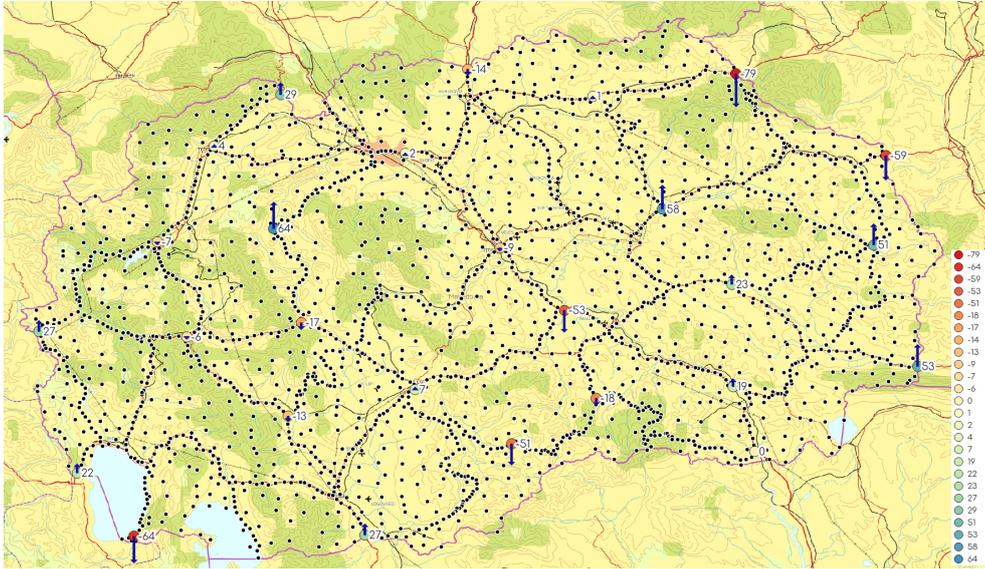


Figure 5: Gravity data (black dots) and as circles a comparison between quasigeoid model to GNSS/Leveling data (H_{nvt1}). The deviations are given in three ways; by colors of the circle, an arrow and a number (in mm) beside the circle.

| Point | Latitude | Longitude | H_{nvt1} [m] | Difference [cm] |
|--------|----------|-----------|----------------|-----------------|
| V5-R5 | 41.98916 | 22.87295 | 1147.38 | -9.1 |
| MJ01 | 40.91886 | 20.77089 | 783.00 | -6.5 |
| MJ58 | 42.21898 | 22.45438 | 1144.81 | -6.4 |
| 253 | 41.17868 | 21.82642 | 806.55 | -6.1 |
| FR1159 | 41.55437 | 21.97394 | 137.93 | -4.6 |
| N26D | 41.30697 | 22.06335 | 856.73 | -2.9 |
| N12 | 41.51957 | 21.23839 | 534.90 | -2.4 |
| 134 | 41.25671 | 21.20177 | 632.16 | -2.2 |
| 104 | 42.23186 | 21.70375 | 392.31 | -1.3 |
| N17 | 41.33088 | 21.55709 | 674.85 | -1.1 |
| N10 | 41.73033 | 21.79364 | 264.04 | -0.6 |
| N14 | 41.47926 | 20.91851 | 638.05 | -0.3 |
| L1-R38 | 41.74605 | 20.83615 | 826.89 | 0.1 |
| N3 | 42.15636 | 22.05350 | 589.35 | 0.2 |
| FR1158 | 41.15717 | 22.50682 | 57.27 | 0.9 |
| L2-R4 | 41.99544 | 21.52931 | 232.22 | 1.1 |
| MJ05 | 41.09724 | 20.61184 | 993.42 | 1.3 |
| V1-R12 | 42.16071 | 21.18076 | 779.26 | 1.5 |
| MJ15 | 42.01631 | 20.99552 | 451.67 | 1.7 |
| N23 | 41.33998 | 22.44567 | 76.88 | 2.8 |
| N27 | 41.62718 | 22.44212 | 393.16 | 3.0 |
| FR1164 | 41.49456 | 20.50522 | 586.61 | 3.7 |
| MJ19 | 40.92269 | 21.41646 | 588.71 | 4.5 |
| N21 | 41.73809 | 22.83897 | 843.55 | 4.5 |
| L17-25 | 41.78394 | 21.16144 | 539.20 | 5.0 |
| MJ37 | 41.39654 | 22.96152 | 191.87 | 6.2 |
| N7 | 41.83841 | 22.24816 | 304.22 | 7.1 |

Table 4: Differences between nmac.v2_xgm compared to 27 updated GNSS/Leveling points of the height system H_{nvt1} .

| Point | Latitude | Longitude | H_{normal} [m] | Difference [cm] |
|--------|----------|-----------|------------------|-----------------|
| MJ58 | 42.21898 | 22.45438 | 1144.49 | -8.3 |
| MJ01 | 40.91886 | 20.77089 | 782.68 | -7.5 |
| 253 | 41.17868 | 21.82642 | 806.24 | -6.9 |
| 104 | 42.23186 | 21.70375 | 391.99 | -5.8 |
| N3 | 42.15636 | 22.05350 | 589.04 | -4.7 |
| 134 | 41.25671 | 21.20177 | 631.86 | -2.4 |
| V5-R5 | 41.98916 | 22.87295 | 1147.06 | -2.0 |
| N26D | 41.30697 | 22.06335 | 856.40 | -1.7 |
| N17 | 41.33088 | 21.55709 | 674.54 | -1.4 |
| N12 | 41.51957 | 21.23839 | 534.57 | -1.2 |
| MJ19 | 40.92269 | 21.41646 | 588.43 | -1.0 |
| FR1159 | 41.55437 | 21.97394 | 137.54 | -0.6 |
| L2-R4 | 41.99544 | 21.52931 | 231.87 | -0.2 |
| N10 | 41.73033 | 21.79364 | 263.67 | 1.1 |
| N27 | 41.62718 | 22.44212 | 392.82 | 1.1 |
| N14 | 41.47926 | 20.91851 | 637.71 | 1.3 |
| N7 | 41.83841 | 22.24816 | 303.91 | 1.5 |
| MJ05 | 41.09724 | 20.61184 | 993.10 | 1.9 |
| FR1164 | 41.49456 | 20.50522 | 586.29 | 2.2 |
| L1-R38 | 41.74605 | 20.83615 | 826.55 | 2.5 |
| MJ37 | 41.39654 | 22.96152 | 191.55 | 3.3 |
| V1-R12 | 42.16071 | 21.18076 | 778.92 | 3.8 |
| FR1158 | 41.15717 | 22.50682 | 56.91 | 3.9 |
| MJ15 | 42.01631 | 20.99552 | 451.32 | 3.9 |
| N23 | 41.33998 | 22.44567 | 76.52 | 4.8 |
| N21 | 41.73809 | 22.83897 | 843.21 | 5.5 |
| L17-25 | 41.78394 | 21.16144 | 538.86 | 7.0 |

Table 5: Differences between nmac.v2_xgm compared to 27 GNSS/Leveling points of the height system H_{normal} .